

# Generation of high-purity higher-order Laguerre-Gauss beams at high laser power

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We have investigated the generation of highly pure higher-order Laguerre-Gauss (LG) beams at high laser power of order 100 W, the same regime that will be used by 2<sup>nd</sup> generation gravitational wave interferometers such as Advanced LIGO. We report on the generation of a helical type LG<sub>33</sub> mode with a purity of order 97 % at a power of 83 W, the highest power ever reported in literature for a higher-order LG mode.

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**Introduction** The generation of Laguerre Gauss (LG) optical beams has significantly gained interest in recent times. LG modes present in fact several unusual features that make them suitable for a wide range of applications. In physics for example, *donut-shaped* LG beams confine particles in optical traps [1, 2] or speed-up charged particles in particle accelerators [3]; higher-order *multi-ringed* LG beams form toroidal traps for Bose-Einstein condensates [4]; LG beams act as *optical spanners* transferring their orbital angular momentum to spin macroscopic particles [5]. In the last years, use of LG beams has been reported in the most diverse areas of science, some examples are material processing [6], microscopy [7], lithography [8], motion sensors [9], biology [10], biomedics [11].

Higher-order helical type LG modes have been also proposed as upgrades to the readout beams of 2<sup>nd</sup> generation gravitational wave (GW) interferometers such as Advanced LIGO [12] and Advanced VIRGO [13], and are currently baselined for the Einstein Telescope [14]. The wider, more uniform transverse intensity distribution of a subset of these beams, compared to the currently used LG<sub>00</sub> fundamental mode, can effectively average over the mirror surface fluctuations, to mitigate the effects of brownian motion of the mirror surfaces on the detector GW sensitivity [15–17]. Using LG modes can also lead to a reduction of thermal effects such as distortions in the mirror substrates, when operating at the high laser power regime envisioned for these detectors [18]. Theoretical studies have initially proven the compatibility of LG modes with the control schemes commonly employed, and identified the LG<sub>33</sub> beams as a good trade-off between mirror thermal noise suppression and beam clipping losses [19]. Subsequent laboratory experiments have then demonstrated the generation of LG modes at the required purity, and the possibility of implementing interferometric measurements using LG beams [20, 21].

One crucial step into a realistic implementation of LG modes in GW interferometers is to demonstrate the generation of such beams at the high power levels of order 100 W foreseen by next generation detectors. High power LG beams should also comply with the stringent require-

ments that current GW laser sources have successfully achieved, and present comparably high levels of purity, stability and low noise [22–24]. Generation of LG beams of order tens of W has been reported in literature [25–28], although for different types of applications and limited to lower order *donut-shape* LG<sub>01</sub> beams only. These beams do not meet the requirements discussed above and furthermore are based on beam shaping techniques which have little adaptability and are hardly exportable to higher-order modes or to more generic applications.

We have investigated the generation of higher-order LG modes at the high laser power regime required for operating 2<sup>nd</sup> generation GW interferometers at full sensitivity. The experiment is based on a beam preparation method originally developed by some of the authors at low power [20] and potentially scalable to full scale interferometers. Our investigation aimed not only to generate higher-order LG beam at the highest possible laser power, mode purity and conversion efficiency, but also to identify potential limits of the technology. In this letter we present our experiment's details and discuss the results. We stress that, due to the simplicity of the experimental scheme, this method is adaptable to a variety of applications, therefore the presented results are potentially relevant to a broader audience than the GW community. **LG modes:** LG modes are a complete and orthogonal set of solutions for the paraxial wave equation. The complex amplitude of a helical type LG<sub>pl</sub> mode, with radial and azimuthal indices  $p$  and  $l$ , is usually described as [29]:

$$\text{LG}_{pl}^{\text{hel}}(r, \phi, z) = \frac{1}{w(z)} \sqrt{\frac{2p!}{\pi(|l|+p)!}} \left( \frac{\sqrt{2}r}{w(z)} \right)^{|l|} L_p^{|l|} \left( \frac{2r^2}{w^2(z)} \right) \times e^{i(2p+|l|+1)\Psi(z)} e^{\left(-\frac{ikr^2}{2R_c(z)} - \frac{r^2}{w^2(z)} + il\phi\right)}. \quad (1)$$

Here  $(r, \theta, z)$  are cylindrical-polar coordinates,  $k$  is the wavenumber,  $w(z)$  the beam radius,  $R_c(z)$  the radius of curvature of the beam wavefront,  $\Psi(z)$  the Gouy phase, and  $L_p^{|l|}(x)$  are the generalised Laguerre polynomials. LG beams are axisymmetric and have spherical wavefront, so they are natural eigenmodes of optical systems whose op-

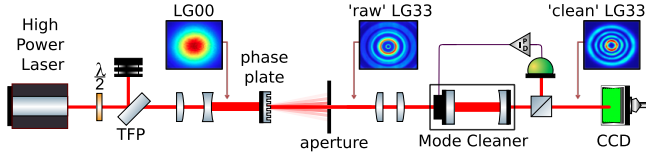


Figure 1: Cartoon of the experimental setup described in the paper. The main components are labelled. Beam dumps, steering mirrors, waveplates and polarisers are not shown.

tical surfaces are spherical and whose symmetry is cylindrical. The order of a LG mode is given by the number  $(2p + |l|)$ : when circulating in optical resonators, modes of the same order experience same resonance conditions, due to the  $e^{[i(2p+|l|+1)\Psi(z)]}$  phase term, so the cavity is degenerate for this family of modes. The  $L_p^{|l|}(x)$  term is what gives LG modes their characteristic *ringed* shape, while the azimuthal phase dependence  $e^{il\phi}$  is responsible for their orbital angular momentum,  $l\hbar$  per photon.

**The experiment** A sketch of the experimental setup is presented in Fig. 1. A high power, ideally pure fundamental mode laser beam is mode-matched to a desired waist size via a telescope, and then sent on a diffractive phase plate, an etched glass substrate with varying thickness which can imprint the LG<sub>33</sub> spiralling phase pattern onto the wavefront of the input beam. The diffraction orders are separated with an aperture, and the main diffracted beam, a composite beam with a dominant LG<sub>33</sub> over a background of higher-order modes of minor intensity, is then injected to a linear mode cleaner (MC) cavity, which is alternatively used to analyse the mode content of the input beam itself (scan mode) or to *spatially* filter out non order 9 LG modes (locked mode) to enhance the mode purity of the LG<sub>33</sub> beam generated in transmission. This is eventually recorded by means of a high dynamic range photodiode, for measurement and control purposes, and by a CCD camera, for off-line analysis. Light power measurements are performed at different stages along the optical setup, namely before and after the phase plate, at the MC input and, when the MC cavity is locked, both in reflection and in transmission from the cavity. Similarly, images of the beam intensity distributions are taken at analogous positions for mode content analyses.

The high-power laser source is the Reference System for the Advanced LIGO pre-stabilised laser (PSL) [22–24] and it is located at the Hannover labs where this experiment was performed. The PSL consists of a 2 W Nd:YAG non-planar ring oscillator, two stages of amplification (up to 35 W and 200 W respectively), and a ring cavity at the output, which provides filtering for beam’s spatial profile, pointing and power fluctuations. The output is a 140 W, 1064 nm, continuous wave, 99.5% pure LG<sub>00</sub> beam.

The phase plate mode conversion method [30] was chosen amongst other successful techniques [31–33] for the compatibility of passive glass components with the high

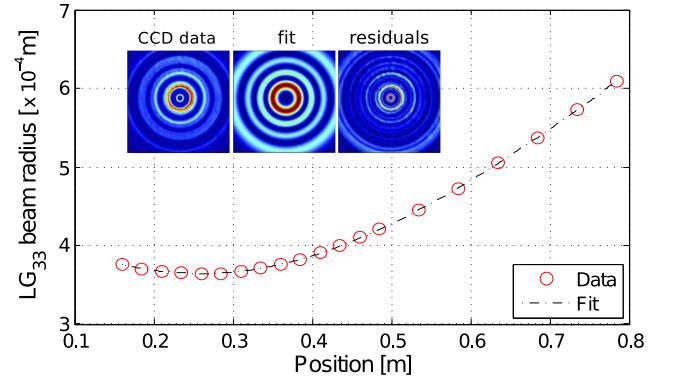


Figure 2: Profile of the LG<sub>33</sub> beam injected into the MC cavity, with best fit shown for comparison. The insets show an example of fitting of a LG<sub>33</sub> beam, with the intensity patterns of a measured beam compared to the fit and related residuals.

power regime to be tested in this experiment, and for the relative simplicity of implementation. Our phase plate is a 3 mm thick fused silica substrate with  $3000 \times 3000$ ,  $7 \mu\text{m}$  side etched pixels, with 8 levels of etching depth resolution [37]. The etched grating phase pattern reproduces the spiralling helical LG<sub>33</sub> mode phase structure. On top, a 2.3 mrad angle blazed grating pattern is superimposed to separate the main diffraction order beam from unmodulated residuals of the LG<sub>00</sub> mode [32]. FFT beam propagation methods and modal analysis of the beam in the far field were used to estimate the efficiency of this phase plate design in the conversion from LG<sub>00</sub> to LG<sub>33</sub>, which is in the region of 75% depending on the correct size and relative alignment of the incident beam with respect to the phase plate itself [34]. To avoid having light reflected towards the laser, a 1064 nm anti-reflective coating was deposited on both surfaces of the phase plate. Measurements showed that about 95% of the light power successfully transmits into the main diffraction order beam, about 4% is dispersed in higher diffraction orders and less than 0.2% is reflected towards the laser source.

The MC is a 21 cm long, plano-concave linear cavity, with 1" fused silica mirrors glued to the ends of a rigid Al spacer. Highly-reflective coatings ( $R = 97.5\%$ ) were deposited on the mirror substrates in a single coating run, aiming for a nominally impedance matched, maximised transmission cavity. The MC has stability parameter  $g \approx 0.8$ , Free Spectral Range = 714 MHz, measured finesse  $F \approx 130$ . Its microscopic length is controlled via a PZT located between the spacer and the input mirror. The error signal for the feedback control is generated by dithering the input mirror position with the PZT and then extracted from the light transmitted by the cavity.

Mode matching of the LG beam generated by the phase plate to the MC eigenmode is non trivial and proved crucial to the successful operation of the cavity. Since conventional beam profilers do not usually resolve LG

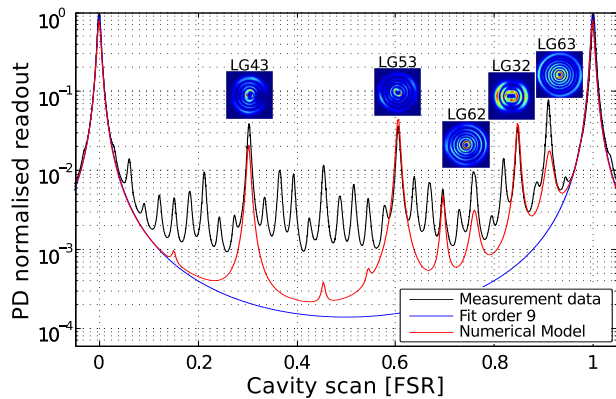


Figure 3: Light power transmitted by the MC measured as a function of the cavity length (black line). The resonant peaks at 0 and 1 FSR are order 9 modes, nominally  $LG_{33}$ . The fit to the ‘ $LG_{33}$ ’ peak measured data is shown for comparison (blue). The red curve is derived with a numerical model that assumes the following mode power distribution: 75% in  $LG_{33}$ , 8% in  $LG_{63}$ , 4% in  $LG_{43}$ ,  $LG_{53}$  and  $LG_{32}$ , 1% in  $LG_{62}$ . The insets show CCD images of these non order 9 modes.

modes, we first recorded the beam intensity profile with a CCD camera placed along the beam path, then the images were analysed using customised fitting scripts [35] which automatically identify the dominant  $LG_{33}$  mode and estimate the beam radius at the given position. Subsequent adjustments of the lenses rapidly led to match the beam waist parameters to within few  $\mu\text{m}$  from the aimed value, in this case  $w_0=365\mu\text{m}$ . In full scale GW interferometers, acceptable matching errors are of order 1%. Our result shows that mode matching of higher-order LG beams can be performed with comparable accuracy. An example of this analysis is given in Fig. 2.

We used measurements of the light transmitted by the MC as a function of the cavity length (*cavity scans*) to investigate the mode content of the beam produced by the phase plate, as in the example in Fig. 3. The relevant non order 9 modes were first identified via inspection of the CCD images, then their amplitude, usually a few % of the total power, and the exact mode content of the overall beam could be reproduced with and compared to numerical simulations [36]. On average, the fraction of the beam power in order 9 modes is  $(75 \pm 5)\%$ , in agreement with the FFT model prediction [34].

**Results** The measurement procedure described above was repeated at progressively increasing input laser power, until the maximum available power was injected on the phase plate. Increasing the laser power stepwise allowed not only for a prevention of damage caused by high powers but also for identifying the potential rise of power-dependent dynamics and potential shortcomings from thermal effects or intra-cavity beam distortions.

We show the main results of this experimental campaign in Fig. 4, where we plot the light power measured

at different locations along the setup as a function of the incident  $LG_{00}$  beam power. First, the linear response of the power transmitted from the phase plate indicates that no effects such as light absorption are arising in the phase plate as the power scales up. We also plot for completeness the same beam when it is propagated to the input of the MC. The 7% reduction in power is consistent with losses likely arising in the intermediate auxiliary optical components and with uncertainties in the measurement calibration [38]. The most notable results in Fig. 4 are the measurements of the light power reflected and transmitted by the MC when this is resonant to order 9 modes. Also in this case the system response is largely linear: the MC length could be locked to the resonance up to full power, for a maximum 83 W *clean*  $LG_{33}$  mode transmitted from the MC when a 122 W *raw* LG beam was injected at input. To identify potential power-dependent degradations in the mode content of the generated beam, cavity scan analyses were made at every laser power level. The non order 9 content increased by no more than 5% at maximum power, confirming that expected heating processes are arising in some component of the beam generation path, however at a scale which is reasonably small for this type of setup. Even so, the structure of the  $LG_{33}$  output beams did not degrade up to the highest power levels, as shown in the example in Fig. 5 where we plot the intensity profile of the 83 W transmitted by the MC.

We assess the purity of the *clean*  $LG_{33}$  beam as the fraction of power in the beam which is in the desired mode and estimate it via the squared inner product  $\langle LG_{33} | \sqrt{I_{\text{meas}}} \rangle^2$  between the theoretical  $LG_{33}$  amplitude distribution and the one measured with the CCD camera,  $\sqrt{I_{\text{meas}}}$  [39]. Results are shown in Fig. 6 (top) as a function of the correspondent  $LG_{33}$  beam power. Over the range of investigation, the  $LG_{33}$  mode purity is above 95%, and no clear trend or degradation is observed. In

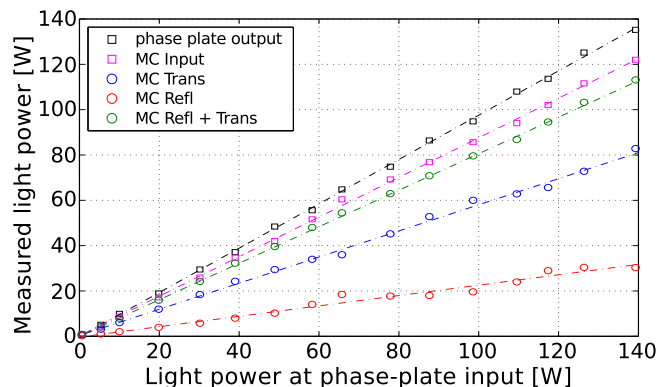


Figure 4: Measurement of the light power at different locations in the setup as a function of the injected laser power. Statistical uncertainties in the measurement data are smaller than the marker’s size and here not reported. Systematic errors in the calibration of each power curve is of order 5%.

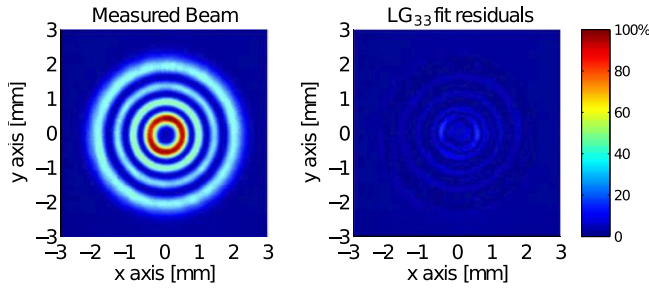


Figure 5: Intensity profile of the 83 W  $LG_{33}$  beam transmitted by the MC cavity (left) compared with fit residuals (right). Maps have same units and scale.

Fig. 6 (bottom) we show the fraction of the injected light power which is transmitted by the resonant MC cavity. On average, 68% is transmitted into a *pure*  $LG_{33}$  beam, for a  $LG_{33}$  MC cavity throughput about 90%. Also here, no trend can be observed. Taking into account losses in the rest of the apparatus, the overall  $LG_{00}$  to  $LG_{33}$  conversion efficiency is about 59%.

**Summary and conclusions** Our experimental investigation into the generation of higher-order LG beams at high laser power proved successful: a 83 W  $LG_{33}$  beam with purity of order 97% was obtained from a 138 W  $LG_{00}$  laser beam, sent through a phase plate and a linear cavity. To our knowledge at the time of writing, this is the highest power ever reported for a higher-order LG beam. As a by-product, we have also shown that profiling of LG beams can be performed at the same level of accuracy commonly achieved with  $LG_{00}$  beams.

The beam generation method seems viable for high power applications. The system response was mostly linear over the entire range of investigation. The conversion efficiency, here partly limited by losses in auxiliary optics, can be easily improved with an engineered design of the conversion apparatus, up to a maximum set by the conversion efficiency of the phase plate design. Stability and noise performances were not investigated in this study.

In this work, we have described a method to create a user-defined LG mode from a highly stable, high-power laser. We have successfully demonstrated that this technique creates modes of high purity with a good conversion efficiency and is compatible with common setups used for the laser pre-stabilisation and injection to GW interferometers. This is an important step towards demonstrating technical readiness of LG modes for use in high-precision interferometry and in particular for future GW detectors such as the Einstein Telescope, as well as for the many other areas of science and technology where LG modes have recently found successful application.

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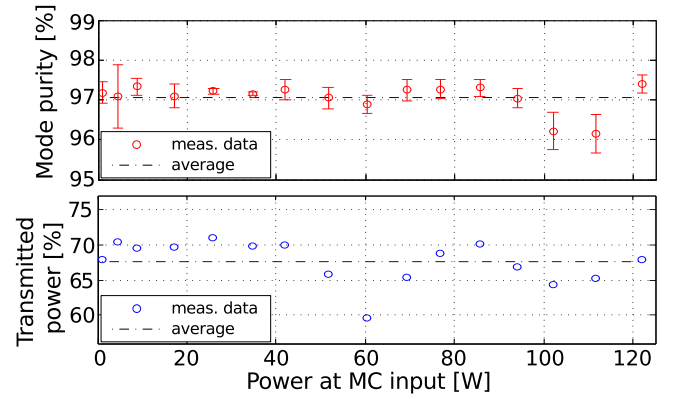


Figure 6: Top:  $LG_{33}$  purity of the beam transmitted by the mode cleaner. Bottom: fraction of the injected light power which is transmitted from the resonant mode cleaner (circles). Dashed lines show average values. Statistical errors are negligible, while calibration uncertainty is here of order 7%.

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- [37] The phase plate was manufactured by Jenoptik GmbH based on a custom design by some of the authors [34].
- [38] The error in the calibration of each measurement curve is of order 5% and depends on the beam size at the specific measurement position, on the auxiliary components utilised in each case and on the instruments themselves.
- [39] We note that since the CCD measures beam intensities, this measurement is in principle degenerate for beams with radial index  $p = 3$  and azimuthal indices  $l = \pm 3$ .